# COMPLEX ANALYSIS QUALIFYING EXAM

Show all your work and explain all your reasoning. You may use any standard results, as long as you state clearly what results you are using. Exception: you may not use a result which is the same as the problem you are being asked to do. Each problem has a noted value, in total 40 points.

1. (10 points) Let $f$ be a holomorphic function in the punctured unit disk $D \setminus \{0\}$. Suppose

$$|f(z)| \leq \frac{1}{\sqrt{|z|}}$$

for all $z \in D \setminus \{0\}$. Is the singularity of $f$ at 0 removable, a pole, or essential? Justify your answer.

2. (10 points) Evaluate the integral

$$\int_0^\infty \frac{x^2 dx}{1 + x^4}.$$

3. (10 points) Let $U$ be an open convex subset in