ALGEBRA QUALIFYING EXAM FALL 2018

Exercise 1. Suppose p is a prime. Show that the Galois group of $x^5 \ \ 1 \in \mathbb{F}_p[x]$ depends only on $p \pmod 5$, and compute it for each congruence class of $p \pmod 5$.

Exercise 2. Let R be a Dedekind domain with eld of fractions K Show that for any two proper fractional ideals I, J there are $\ , \ \in K$ with $\ I, \ J \ \ R$ integral and $\ I + \ J = R$.

Exercise 3. Suppose that R is a Noetherian ring and $\mathfrak{p} \ \ R$ is a prime ideal such that $R_{\mathfrak{p}}$ is an integral domain. Show that there is an $f \in R \setminus \mathfrak{p}$ such that R_f is an integral domain where R_f